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TECHNICAL NOTE

A nonequilibrium algebraic model for turbulent density fluctuations

H. WEI† and C. P. CHEN‡
University of Alabama in Huntsville, Huntsville, AL 35899, U.S.A.

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INTRODUCTION

Advanced hypersonic flight vehicles generally utilize a window concept to protect an optical or radar guidance sensor. This aero-window is formed by passing a high-speed gas jet across the front of the lasering cavity so that the beam exits roughly in a direction perpendicular to the direction of the gas flow. The compressible turbulent shear layer formed over optical windows can cause severe limitations to target detection and recognition, because the turbulent fluctuations especially density fluctuations will lead to the index-of-refraction fluctuations [1]. For providing the exact location of the object of interest, one must predict the optical degradation caused by index-of-refraction fluctuation, or essentially, predict the intensity of density fluctuations in compressible turbulent shear layers.

Lutz [2] derived a density fluctuation model from the differential form of the Crocco–Busemann temperature solutions. This model neglected the pressure fluctuation in supersonic boundary layers which was shown to be significant by the experimental data of Laderman and Demetriades [3]. A turbulent transport equation for the variance of index-of-refraction fluctuations was derived by Smith *et al.* [4] based on Spalding's scalar fluctuation transport equation [5]. The empirical constants used in ref. [4] were chosen to be those values used for the variance of temperature fluctuations. Bogdanoff [6] obtained an equilibrium model by neglecting the convection term and diffusion term of the scalar transport equation. This resulted in a mixing-length type equilibrium algebraic equation for scalar fluctuation quantity. The equilibrium assumption is only valid for the very near wall region, where the generation and destruction terms far outweigh other terms in the equation. Thus, taking into account the convection and diffusion terms was expected to provide significantly improved predictions over Bogdanoff's equilibrium model in flows in which the convection and diffusion are not negligible. Development of a nonequilibrium algebraic model based on the second-order algebraic Reynolds stress model is the subject of this Note.

THE ALGEBRAIC MODEL FOR SCALAR QUANTITY FLUCTUATIONS

The fluctuations of density or temperature can be treated as passive scalar having no effect on the mean flow [5]. Therefore, the equation of scalar quantity fluctuations is uncoupled from the governing equations. In some cases, the scalar quantity is not passive and does affect the mean flow. For example, the effects of density fluctuations upon the mean flow are known to be significant for flows which are highly buoyant or for flows with large density differences induced by passing shocks. This situation is not considered in this study. The expression relating the mean-square random phase error and the index-of-refraction fluctuations across the shear layer is given as [8]

$$\overline{\phi^2} = 2K^2 \int \overline{n'n'}\Lambda \, dy, \tag{1}$$

where K is the wave number of coherent radiation, n' is the fluctuation of index-of-refraction, Λ is the length scale of the index-of-refraction fluctuations. The index-of-refraction may be related to the density through the Gladstone–Dale relation

$$n = 1 + \beta_m \frac{\rho}{\rho_{ref}}, \tag{2}$$

where β_m is the Gladstone–Dale constant for the mixture. So, for correcting the random phase error and improving the optical quality of the laser beam, the accurate prediction of the density fluctuations is the key issue.

Let g represent the intensity of density fluctuation $\sqrt{\overline{\rho'\rho'}}$, then the transport equation of g in the boundary layer can be derived (cf. refs. [4, 5]) and the modeled equation is given as

$$\underbrace{\rho\frac{Dg}{Dt}}_{\text{Convection}} = \underbrace{\frac{\partial}{\partial y}\left(\frac{\mu+\mu_t}{\sigma_g}\frac{\partial g}{\partial y}\right)}_{\text{Diffusion-}D_g} + \underbrace{C_{1g}\mu_t\left(\frac{\partial\rho}{\partial y}\right)^2}_{\text{Production-}P_g} - \underbrace{C_{2g}\frac{\rho g \varepsilon}{k}}_{\text{Dissipation-}\varepsilon_g}, \tag{3}$$

where $C_{1g} = 2.8$, $C_{2g} = 1.4$. In the equilibrium model [6], the convection and diffusion terms of the transport equation for g are neglected and the production and dissipation terms are set equal. The result is

$$g = \frac{C_{1g}}{C_{2g}}\frac{k}{\rho\varepsilon}\mu_t\left(\frac{\partial\rho}{\partial y}\right)^2. \tag{4}$$

Utilizing similar ideas proposed for ASM (algebraic stress model for turbulence Reynolds stresses) by Rodi [8], the difference of the convection term and diffusion term of the g transport equation can be related to that of the turbulent kinetic energy (k) equation as

$$\rho\frac{Dg}{Dt} - D_g = \frac{g}{k}\left(\rho\frac{Dk}{Dt} - D_k\right). \tag{5}$$

In boundary layers, this second-order based nonequilibrium model has the form

† Currently with Engineering Sciences Inc.

‡ Author to whom correspondence should be addressed.

NOMENCLATURE

C_{1g}	model constant in the scalar-fluctuation equation
C_{2g}	model constant in the scalar-fluctuation equation
g	turbulent density fluctuation intensity
k	turbulent kinetic energy
M_e	free stream Mach number
M_t	local turbulence Mach number
U	density-weighted mean axial velocity
y	lateral direction.

Greek symbols

δ	boundary layer thickness from experiments
ε	dissipation rate of turbulent kinetic energy
μ, μ_t	molecular viscosity and turbulent eddy viscosity
ρ, ρ	mean and fluctuating density
σ_g	turbulent Schmidt number.

$$C_{1g}\mu_t\left(\frac{\partial\rho}{\partial y}\right)^2 - C_{2g}\frac{\rho g\varepsilon}{k} = \frac{g}{k}\left(\mu_t\left(\frac{\partial U}{\partial y}\right)^2 - \rho\varepsilon\right). \quad (6)$$

After rearrangement, we get the final algebraic form

$$g = \frac{C_{1g}\mu_t\left(\frac{\partial\rho}{\partial y}\right)^2 k}{\mu_t\left(\frac{\partial U}{\partial y}\right)^2 + (C_{2g}-1)\rho\varepsilon}. \quad (7)$$

This derivation is based on the scalar transport equation originally modeled for low speed shear flows. For supersonic and hypersonic boundary layer flows, the augmentation of the dissipation of kinetic energy (ε) in compressible turbulence due to its dilatational components is found necessary [9]. The extra 'production' of ε is modeled by making the model coefficient dependent on the local Mach number. By performing numerical experiments on the standard boundary layer flows, this algebraic model for density fluctuation is modified as

$$g = \frac{C_{1g}\mu_t\left(\frac{\partial\rho}{\partial y}\right)^2 k}{\mu_t\left(\frac{\partial U}{\partial y}\right)^2 + (C_{2g}-1)(1+F_{mt})\rho\varepsilon}. \quad (8)$$

where

$$F_{mt} = 24M_t^{0.25} \quad (9)$$

$M_t = \sqrt{k}/a$ is the turbulence Mach number, a is the speed of sound.

RESULTS AND DISCUSSION

The current nonequilibrium model is evaluated using several sets of the experimental data of density fluctuation intensity in high speed boundary layer flows. The GASP (general aerodynamic simulation program) code [10] was used to solve the fluid flow equations. The density-weighted averaged velocity fields, as well as the flow field turbulent kinetic energy (k) and the turbulent kinetic energy dissipation rate (ε) obtained from the GASP solutions were used to compute turbulent density fluctuation intensities according to equation (8). A space marching technique was applied with 81 grid points across the boundary layers. Grid-independent results were assured by using very fine grid meshes near the wall regions. Detailed numerical methods and implementations can be found in refs. [10, 11].

The supersonic adiabatic-wall boundary layer cases are calculated first. Figures 1 and 2 show the comparisons of computed density fluctuation profiles using the current non equilibrium model to the experimental data ref. [12]. Predictions using the equilibrium model [6] as well as Lutz's results are also shown for references. The data and predicted values shown a maximum in the middle portion of the boundary layer and the density fluctuation intensity increase with larger free stream Mach number M_e. The current model

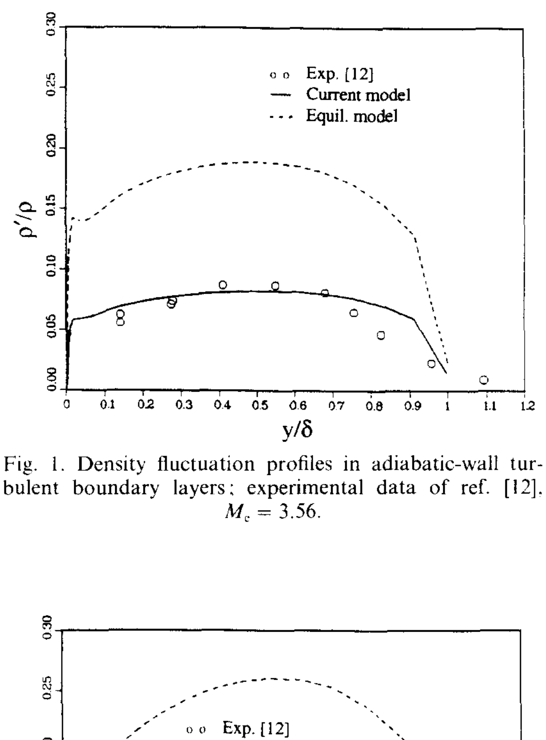

Fig. 1. Density fluctuation profiles in adiabatic-wall turbulent boundary layers; experimental data of ref. [12], $M_e = 3.56$.

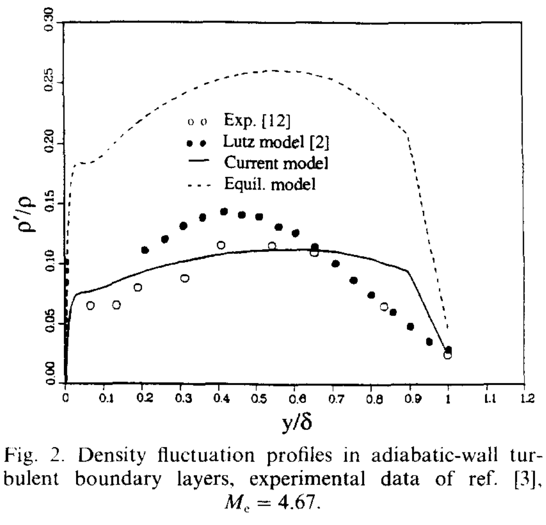

Fig. 2. Density fluctuation profiles in adiabatic-wall turbulent boundary layers, experimental data of ref. [3], $M_e = 4.67$.

adequately predicts the quantitive behaviour of density fluctuation intensity with in the boundary layers.

The comparisons of the measured data of refs. [3, 13] to computational $\sqrt{\rho'\rho'}/\rho$ profiles inside nonadiabatic supersonic and hypersonic boundary layers by using the nonequilibrium model, the equilibrium model and Lutz's results are shown in Figs. 3 and 4. The results of the current model also agree more favourably with the experimental data, compared with the results obtained by the equilibrium model and Lutz's model. The predicted trend is in qualitative agreement with the data for hypersonic flow (Fig. 4). At the wall region of a hypersonic boundary layer, the viscous dissipation coupled with the aerodynamic heating produced extra dissipation of the turbulent kinetic energy, which cannot be adequately modeled by the low-Reynolds-number two-equation model [14]. This deficiency of kinetic energy profile gives rise to the inaccurate predictions of density fluctuations near the wall region.

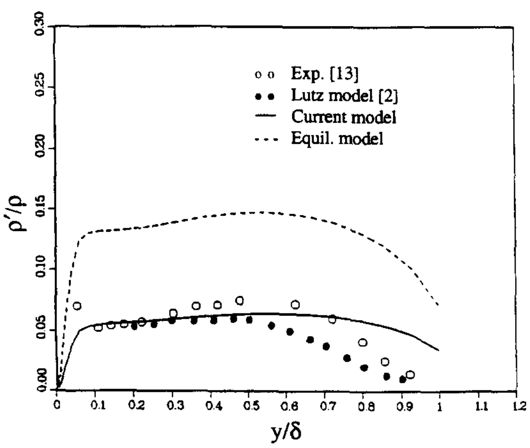

Fig. 3. Density fluctuation profiles in isothermal wall turbulent boundary layers, experimental data of ref. [13], $M_e = 3.0$, $T_w = 226$ K.

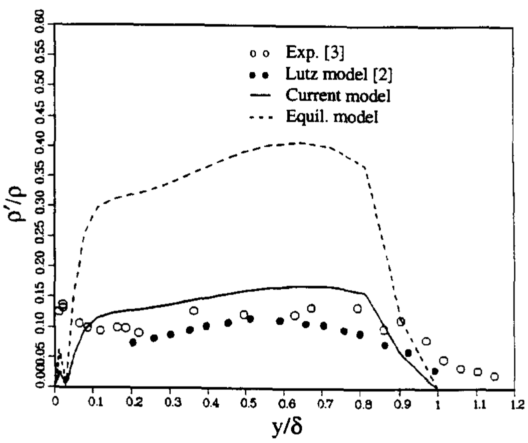

Fig. 4. Density fluctuation profiles in non-adiabatic wall turbulent boundary layers, experimental data of ref. [3], $M_e = 9.4$, $T_w = 304$ K.

CONCLUSION

To correct the sight-to-target degradation caused by the index-of-refraction fluctuation, the density fluctuation must be predicted, because the index-of-refraction fluctuation is strongly related to the density fluctuation. Lutz's model neglected pressure fluctuation that is unsound in supersonic flows. The equilibrium model does not account for the convection and diffusion mechanism of scalar fluctuation transport. The nonequilibrium second-order algebraic model described in this study avoids the equilibrium assumption and accounts for both the history and local effects through an algebraic transport assumption, as seen in equation (5). A series of supersonic and hypersonic flat plate boundary layer flows is utilized to test the new model. The new model shows a significant improvement on the prediction of the density fluctuation intensity over the equilibrium model and Lutz's model.

REFERENCES

1. G. W. Sutton, Aero-optical foundations and applications, *AIAA J.* **23**, 1525–1537 (1985).
2. S. A. Lutz, Modeling of density fluctuations in supersonic turbulent boundary layers, *AIAA J* **27**, 822–823 (1989).
3. A. J. Laderman and A. Demetriades, Mean and fluctuating flow measurements in the hypersonic boundary layer over a cooled wall, *J. Fluid Mech.* **63**, 121–144 (1974).
4. R. R. Smith, C. R. Truman and B. S. Masson, Prediction of optical phase degradation using a turbulent transport equation of the variance of index-of-refraction fluctuations, AIAA paper 90-0250 (1990).
5. D. B. Spalding, Concentration fluctuations in round turbulent free jet, *Chem. Engng Sci.* **26**, 95–107 (1971).
6. D. W. Bogdanoff, The optical quality of shear layers: Prediction and improvement thereof, *AIAA J.* **22**, 58–64 (1984).
7. K. G. Gilbert, Overview of aero optics. *Aero-Optical Phenomena*, Vol. 80, *Progress in Astronautics and Aeronautics* (Edited by K. G. Gilbert and L. J. Otten), pp. 1–9. AIAA, New York, (1982).
8. W. Rodi, A new algebraic relation for calculating the Reynolds stress, *Mech. Fluid ZAMM* **56**, T219–T221 (1976).
9. J. R. Viegas and M. W. Rubesin, Assessment of compressibility corrections to the k-ε model in high-speed shear layers, *AIAA J.* **30**, 2369–2370 (1992).
10. R. W. Walters, D. C. Slack, P. Cinnella, M. Applebaum and C. Frost, A user's guide to GASP, NASA langley Research Center, Hampton, VA, November, (1990).
11. H. Wei, The development and assessment of advanced turbulence models for complex flows, Ph.D. dissertation, University of Alabama in Huntsville, AL (1994).
12. A. L. Kistler, Fluctuation measurements in a supersonic turbulent boundary layer, *Phys. Fluid* **2**, 290–296 (1959).
13. A. J. Laderman and A. Demetriades, Turbulent shear stresses in compressible boundary layers, *AIAA J.* **17**, 736–744 (1979).
14. Y. S. Chen, C. P. Chen and H Wei, Numerical analysis of hypersonic turbulent film cooling flows, AIAA paper 92-2767 (1992).